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6. AUTHOR(S)
Richard D. James and Oscar Bruno (Authors)

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Minnesota
Minneapolis, MN 55415-1226

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13. ABSTRACT *(Maximum 200 words)*

The research was divided into three different areas: (1) Estimation of the overall behavior of strongly heterogeneous composite materials, (2) Optical Properties of diffraction gratings and (3) Asymptotic behavior of a coalescence problem. The research results are summarized in the final report.

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7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:

1. Asymptotic behavior of a coalescence problem; O. Bruno, A. Friedman and F. Reitich, submitted.
2. The effective conductivity of strongly heterogeneous composites; O. Bruno, to appear in Proc. Royal Soc. London A.
3. Taylor series and bounds for the effective conductivity and the effective elastic moduli of multicomponent composites and polycrystals; O. Bruno, to appear in Asympototic Analysis.
4. Effective conductivity and average polarizability of random polycrystals; M. Avellaneda and O. Bruno, Journal of Mathematical Physics **31**, (1990), 2047-2056.
5. Taylor series and the overall properties of composites, O. Bruno, to appear in the Proceedings of the Eighth Army Conference on Applied Mathematics and Computing, 1990.
6. Interchangeability and bounds for the effective conductivity of the square lattice; O. Bruno and K. Golden, Journal of Statistical Physics **61**$\frac{1}{2}$, (1990), 361-382.

7. The effectivt .onductivity of an infinitely interchangeable mixture; O. Bruno, Communications on Pure and Applied Mathematics **43**, (1990), 769-807.
8. The effective conductivity of composites; O. Bruno, to appear in Lectures in Applied Mathematics, AMS, Proceedings of the AMS-SIAM Summer Seminar on Mathematics of Random Media, 1989.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: Oscar Bruno

BRIEF OUTLINE OF RESEARCH FINDINGS

Bruno's research during this period was divided into three different areas:

1. **Estimation of the overall behavior of strongly heterogeneous composite materials**
He considered electrical properties, and in an ongoing project in colaboration with Perry Leo, elastic properties of mixtures of components with very different electrical or elastic properties. These investigations lead to bounds for the effective moduli for mixtures made of cells of very high or very low conductivity inserted in a conducting matrix, and for the overall elastic properties of materials with holes. For some random systems in which the particles sizes are 20 percent away from the touching size, these new bounds determine the effective properties with an uncertainty of 5 percent. These constitute the first rigorous and accurate estimates for effective moduli of strongly heterogeneous mixtures, which had been previously treated by numerical calculations for somewhat restrictive geometries, such as periodic arrays of spheres.

2. **Optical properties of diffraction gratings**
In this investigation, in collaboration with F. Reitich, a new method for solving a class of PDE's of mathematical physics was introduced. The method was subsequently applied to the problem of determining the diffraction of light by an interface between two dielectrics. This method is based on computing the solution to a PDE by variations of the boundary of the domain where the solution is sought. The basic result here, which was conjectured to be false by some authors, is that the solution of, say, Maxwell's equations associated to diffraction problems, is an analytic function with respect to variations of the boundaries. This method was numerically implemented, and gave excellent results for the kinds of commercial gratings used in applications.

3. **Asymptotic behavior of a coalescence problem**
In this work, in collaboration with A. Friedman and F. Reitich, the asymptotic behavior of a new type of aerosol arising in photographic applications was studied. The mathematical model, which leads to a nonlinear Boltzman-like differential equation, can be described as follows. Consider spherical particles of volume x having "paint" on a fraction y of their surface area. The particles are assumed to be homogeneously distributed at each time t, so that one can introduce the density number $n(x,y,t)$. When collision between two particles occurs, the particles will coalesce if and only if they happen to touch each other, at impact, at points which do not belong to the painted portions of their surfaces. Introducing a dynamics for this model, we studied the evolution of $n(x,y,t)$ and, in particular, the asymptotic behavior of the mass $xn(x,y,t)dx$ as $t \to \infty$. It was found that, for large times, the system

reaches a stationary state with all particles painted. Furthermore, the stationary state was shown to depend continuously with respect to variations of the initial conditions.

Richard James and David Kinderlehrer continued development of a new theory of magnetostriction during this period. Beginning with observations of Donald Lord on the domain structure of $Tb_x Dy_{1-x}Fe_2$, James and Kinderlehrer developed a free energy function for this material and studied stable domain patterns. The first indications of this study are that the composite nature (growth twins on {111} planes) of $TbDyFe_2$ may well contribute to its huge magnetostriction. In the experimental literature it is thought that the growth twins compromise the effect. This recent study drew upon their work on micromagnetics which showed that the minimum free energy is not attained for some crystal symmetries, one of the reasons why micromagnetics has met with limited success in the past.